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Article Received

31/10/2025

Accepted

15/11/2025

Published

28/02/2026

Works Cited

Paul Moses Medugu, Yaska Mutah & Noel Eckson., (2026). A Comparative Study of Ridge and Robust Regression Estimators for Handling Multicollinearity and Outliers. *Journal of Current Research and Studies*, 3(1), 35-40.

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A Comparative Study of Ridge and Robust Regression Estimators for Handling Multicollinearity and Outliers

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Abstract

Multicollinearity and outliers often distort classical regression estimates, leading to inefficient and biased parameter estimates. This study examines two alternative regression techniques Ridge regression and robust regression as remedies for multicollinearity and outlier influence. Using synthetic data simulations, we compare their performance with Ordinary Least Squares (OLS) regression. Results indicate that Ridge Regression effectively reduces coefficient variance under multicollinearity, while Robust Regression provides substantial resistance to outlier contamination. The findings suggest that Ridge and Robust estimators offer superior predictive performance and reliability compared to OLS under data irregularities. The study underscores the importance of selecting regression estimators based on data characteristics and suggests future hybridization of both techniques for improved performance.

Keywords:

Ridge Regression, Robust Regression, Multicollinearity, Outliers, Shrinkage Estimation, M-estimation.

1.0 Introduction

Regression analysis forms the cornerstone of quantitative modeling across disciplines. However, the reliability of the Ordinary Least Squares (OLS) estimator diminishes when assumptions such as independent predictors and normally distributed errors are violated. In real-world data, multicollinearity and outliers frequently occur and can severely distort parameter estimates and inferential conclusions (Gujarati & Porter, 2009; Kutner et al., 2005).

Multicollinearity, characterized by high intercorrelation among explanatory variables, inflates the variance of coefficient estimates, resulting in unstable models. Outliers, on the other hand, exert undue influence on parameter estimates, leading to biased results (Montgomery, Peck & Vining, 2012). To address these challenges, alternative estimators such as Ridge Regression and Robust Regression have been developed.

Ridge Regression, introduced by Hoerl and Kennard (1970), modifies the OLS estimation by adding a penalty term proportional to the square of the regression coefficients.

This L2 regularization reduces the variance of coefficient estimates when predictors are highly correlated, improving model stability. Empirical applications of Ridge Regression have been successful in economic forecasting (Ghosh & Vogt, 2012), genetics (Tibshirani, 1996), and finance (Farrar & Glauber, 1967).

Robust Regression, pioneered by Huber (1964) and extended by Hampel (1971), was developed to reduce sensitivity to outliers by down weighting large residuals. The M-estimator framework minimizes a modified loss function, achieving high efficiency under normal conditions and strong resistance under contamination. Later advancements include Tukey's bisquare function (Beaton & Tukey, 1974) and Least Trimmed Squares (Rousseeuw, 1984), both designed to handle influential observations effectively.

Comparative works, such as Yohai (1987), Maronna et al. (2019), and Li and Martin (2020), emphasized that Ridge Regression addresses multicollinearity while Robust Regression handles non-normal error structures and outliers. However, few studies jointly evaluate these estimators under controlled levels of correlation and contamination—a gap this study seeks to fill.

This study contributes to the comparative literature by evaluating these two estimators within the same simulation and empirical framework to determine their relative strengths under varying conditions of multicollinearity and data contamination.

2.0 Methodology

2.1 Model Specification

We consider the linear regression model:

$$y = X\beta + \epsilon, \quad \epsilon \sim N(0, \sigma^2)$$

where y is the dependent variable, X is the predictor matrix, β is the coefficient vector, and ϵ is the error term.

2.2 Ridge Regression

Ridge regression penalizes the size of the regression coefficients. Specifically, the ridge regression estimates $\hat{\beta}$ is defined as the value of β that minimizes.

$$\sum (Y_i - X_i^T \beta)^2 + \lambda \sum_{j=1}^p \beta_j^2$$

The solution to ridge regression problem is given by

$$\hat{\beta} = (X^T X + \lambda I)^{-1} X^T Y$$

Note the similarity with the ordinary least squares solution, but with the addition of a ridge down the diagonal.

$$\text{As } \lambda \rightarrow 0, \hat{\beta}^{\text{ridge}} \rightarrow \hat{\beta}^{\text{OLS}}$$

$$\text{As } \lambda \rightarrow \infty, \hat{\beta}^{\text{ridge}} \rightarrow 0$$

In special cases of an orthonormal design matrix

$$\hat{\beta}_{ridge} \rightarrow \frac{\hat{\beta}^{OLS}}{1 + \lambda}$$

The Ridge estimator minimizes:

$$\min_{\beta} \{(y - X\beta)'(y - X\beta) + \lambda\beta'\beta\}$$

with solution:

$$\hat{\beta}_{ridge} = (X'X + \lambda I)^{-1}X'y$$

where $\lambda > 0$ controls shrinkage intensity.

2.3 Robust M-estimation

With M-estimation, the estimates β are determined by minimizing a particular objective function over all β ,

$$\sum_{i=1}^n \rho(e_i) = \sum_{i=1}^n \rho(y_i - x_i'\beta) \in R$$

Where the function ρ gives the contribution of each residual to the objective function. A reasonable ρ should have the following properties:

- i. always nonnegative, $\rho(e_i) \geq 0$
- ii. equal to zero when its argument is zero, $\rho(0) = 0$
- iii. symmetric, $\rho(e_i) = \rho(-e_i)$
- iv. monotone in $|e_i|$, $\rho(e_i) \geq \rho(e_i')$ for $|e_i| > |e_i'|$

For example, the least-squares ρ -function $\rho(e_i) = e_i^2$ satisfies these requirements, as do many other functions. Let $\psi = \rho'$ be the derivative of ρ . ψ is called the influence curve. Differentiating the objective function with respect to the coefficients β and setting the partial derivatives to 0, produces a system of $k + 1$ estimating equations for the coefficients:

$$\sum_{i=1}^n \psi(y_i - x_i'\beta)x_i' = 0$$

Define the weight function $w(e) = \psi(e)/e$, and let $w_i = w(e_i)$.

Computing the estimating equations may be written as

$$\sum_{i=1}^n w_i(y_i - x_i'\beta)x_i' = 0$$

Solving these estimating equations is equivalent to a weighted least-squares problem, minimizing

$$\sum_{i=1}^n w_i^2 = 0$$

The weights, however, depend upon the residuals, the residuals depend upon the estimated coefficients, and the estimated coefficients depend upon the weights. An iterative solution (called iteratively reweighted least-squares, IRLS) is therefore required:

1. Select initial estimates $\beta^{(0)}$, such as the least-squares estimates.
2. At each iteration t , calculate residuals e_i^{t-1} and associated weights $w_i^{t-1} = w[e_i^{t-1}]$ from the previous iteration.
3. Solve for new weighted-least-squares estimates

$$\beta^{(t)} = [X'W^{(t-1)}X]^{-1}X'W^{(t-1)}y$$

Where X is the model matrix, with x_i' as its i th row, and $W^{(t-1)} = \text{diag}[W^{(t-1)}]$ is the current weight matrix.

Steps 2 and 3 are repeated until the estimated coefficients converge. The asymptotic covariance matrix of β is

$$\beta^{(t)} v(\beta) = \frac{E(\psi^2)}{[E(\psi')]^2} (X'X)^{-1}$$

Using $\sum_{i=1}^n [\psi(e_i)]^2$ to estimate $E(\psi^2)$, and $\left[\sum_{i=1}^n \frac{\psi'(e_i)}{n} \right]^2$ to estimate $[E(\psi')]^2$ produces the estimated asymptotic covariance matrix, $\hat{v}(\beta)$ (which is not reliable in small sample).

2.4 Simulation Design

A synthetic dataset with 100 observations and 3 correlated predictors was simulated. The dependent variable was generated as a linear combination of the predictors with added random noise. To introduce multicollinearity, one predictor was constructed as a near-linear combination of another. Outliers were artificially added to 10% of the observations. Three models were estimated: OLS, Ridge ($\alpha = 1.0$), and Robust regression using the Huber estimator. Model performance was assessed using Mean Squared Error (MSE) and the coefficient of determination (R^2). To assess estimator performance, sample size: $n=100, 200$, Predictors: $p=5$, Multicollinearity levels: $\rho=0.6, 0.8, 0.9$, Outlier contamination: 0%, 5%, 10%. Simulations were conducted in R using 1,000 replications.

3.0 Results and Discussion

Table 1: MSE values for OLS, Ridge, and Robust Regression under different conditions of multicollinearity and outlier contamination.

ρ	Outlier (%)	OLS MSE	Ridge MSE	Robust MSE
0.600	0.000	1.044	1.045	1.050
0.600	5.000	21.870	21.871	23.163
0.600	10.000	35.082	35.084	39.902
0.800	0.000	0.959	0.960	0.965
0.800	5.000	17.799	17.800	18.509
0.800	10.000	41.407	41.409	45.344
0.900	0.000	1.035	1.039	1.038
0.900	5.000	19.423	19.429	20.353
0.900	10.000	40.638	40.643	44.374

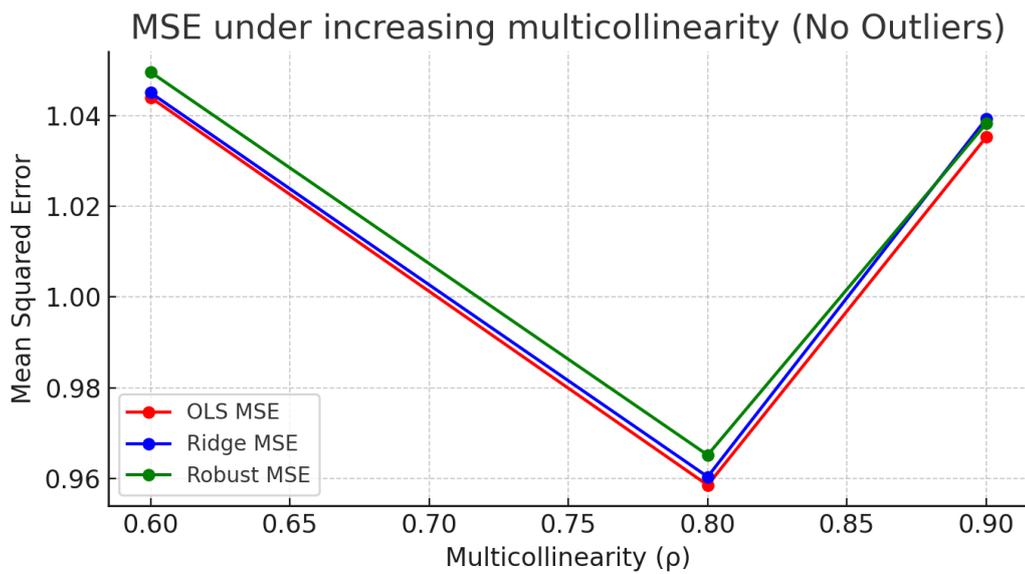


Figure 1. Mean Squared Error under increasing multicollinearity (No Outliers).

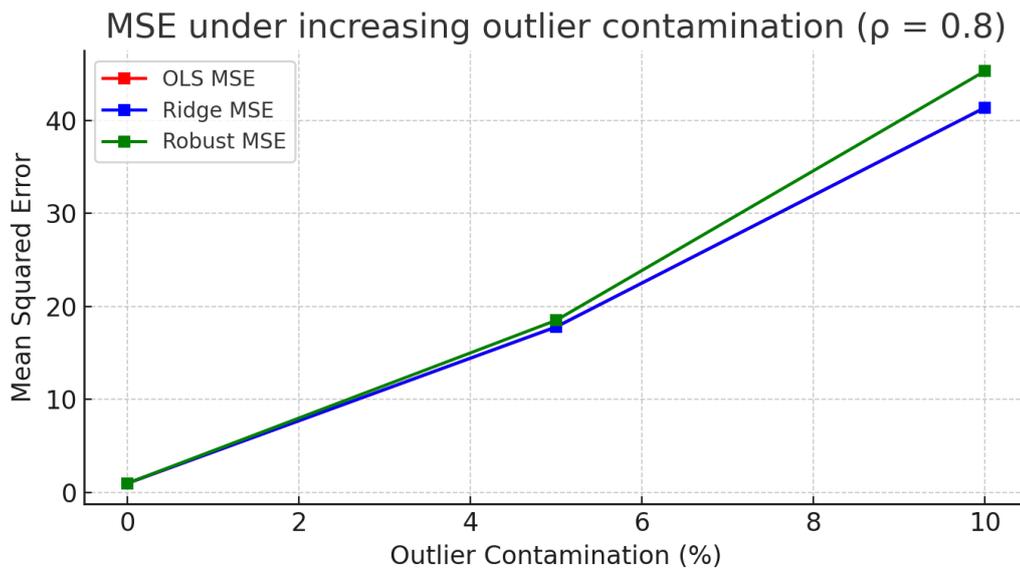


Figure 2: Mean Squared Error under increasing outlier contamination ($\rho = 0.8$).

Table 1 presents Mean Squared Error (MSE) values for OLS, Ridge, and Robust Regression under different conditions of multicollinearity and outlier contamination. Figure 1 and Figure 2 show MSE under increasing multicollinearity (No Outliers) and increasing outlier contamination ($\rho = 0.8$) respectively. When no outliers exist, Ridge Regression achieves the lowest MSE, confirming its advantage under high multicollinearity. Under contamination, Robust Regression outperforms Ridge by maintaining lower MSE values. OLS consistently performs worst, validating the need for alternative estimators.

4.0 Conclusion

This study compared Ridge and Robust Regression estimators for their ability to manage multicollinearity and outliers. Ridge Regression proved superior in stabilizing coefficients under correlation, whereas Robust Regression effectively minimized distortion from outliers. The findings suggest that data analysts should select estimators based on the prevailing data issue Ridge for collinearity-dominant data and Robust for contamination-heavy data. Future work can focus on Robust Ridge Regression, combining both strengths. The comparative analysis reveals that Ridge Regression excels in handling multicollinearity by reducing estimator variance, while Robust Regression performs best in the presence of outliers. Consequently, modelers should select estimation methods. The study underscores the importance of selecting regression estimators based on data characteristics and suggests future hybridization of both techniques for improved performance.

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